The ELISA Test: Stage 4 and 5** Zach Thompson

This is a very complicated question, and I had to read it several times to really understand it properly. I'm still in primary school so I don't know what HIV is, but my mum explained that it is a virus that can make you very poorly because your body is less able to fight off infections, even little ones.

I took another approach which made more sense to me and thought about it backwards. My nanny had alzheimer's disease, and was ill for a very long time.

If there are 100 people and the doctor said that each person had alzheimers, she would be wrong on one occasion. She would be 99% correct, but 1% wrong (a false positive). If the doctor is the test, then she is 99% accurate. But the people she said did have alzheimers, really did. For them, her error rate is virtually nil.

This problem looks at HIV. If you don't have the virus, the ELISA test can incorrectly record a positive result in 1% of patients.

Say 100 HIV negative people are tested:

99 people will correctly receive a negative response or all clear. This means that 99 people were diagnosed 100% correctly.

This leaves 1 person who will incorrectly receive a positive response, even though they don't have it, so 1 person was diagnosed 100% incorrectly.

You have to follow the logic: if you have the virus and the test says you are positive, then you are (you do actually have the virus in your body, the test just tells you this). A positive test result is 100% correct if you are carrying the virus.

But the test is not 100% accurate. If you do not have the virus, out of 100 negative people, one can get a (false) positive result – they are told they have the virus, even though they don't – a 1% error rate.

This is not the same as saying "a positive result means there is a 99% chance of being infected", because if you are carrying the virus, and get a positive result, the test has correctly diagnosed the virus – you already have it in your system.

The question does say that virtually all people with HIV get a positive result and that the probability of a false negative is negligible. The question then goes on to say that in low risk groups, the rate of infection is 1 in 10,000. The figures below are rounded:

	Positive Test Result	Negative Test Result	Total
Person is HIV positive	1 (1 in 10,000)	0 (false negative is negligible)	1
Person is HIV negative	100 (1% of 9,999)	9,899 (99% of 9,999)	9,999
Total	101	9,899	10,000

So the probability of testing positive for HIV and actually having the virus is: $\frac{1}{101} = 0.0099$

This really surprises me because I didn't realise that there could be so many false positives with a medical test. This could make people feel scared. But I'm sure they must have a follow-up check to deal with this. The test is only very accurate for the part of the population with HIV. It is still accurate for the rest of the population, just not 100%.

In a low risk group, for every person with the virus who gets a positive result, there are 100 who are really negative, but the test says they have it (false positive):

1:100

If there is a 1 in 10,000 chance of HIV in a low risk group and the whole world was low risk (which it is not), based on a 7 billion population, there would be 700,000 people with HIV. The approximate number of false positives would be:

$$7{,}000{,}000{,}000 - 700{,}000 \times \frac{1}{100} = 69{,}993{,}000 \approx 70 \text{ million false positives worldwide}$$

There are many more cases of HIV, so this is just hypothetical.

Because some places of the world have a much higher risk, the ratio of 1:10,000 will alter (more people will have the virus). For example, if we stick with our figure of 10,000 but increase infection to 5 in 10,000:

	Positive Test Result	Negative Test Result	Total
Person is HIV positive	5 (5 in 10,000)	0	5
Person is HIV negative	100 (rounding)	9,895	9,995
Total	105	9,895	10,000

Before the ratio of true positive to false positive was 1:100, now it is 5:100 or 1:20, which is worse, because more people have the disease (the number of false positives remain the same, a 1% chance)

The samples are too similar, so I leapt to an infection rate of 200 in 10,000:

	Positive Test Result	Negative Test Result	Total
Person is HIV positive	200 (200 in 10,000)	0	200
Person is HIV negative	98 (1% of 9,800)	9,702	9,800
Total	298	9,702	10,000

Now the ratio of true positives to false positives is reversed… 200:98 or 100:49

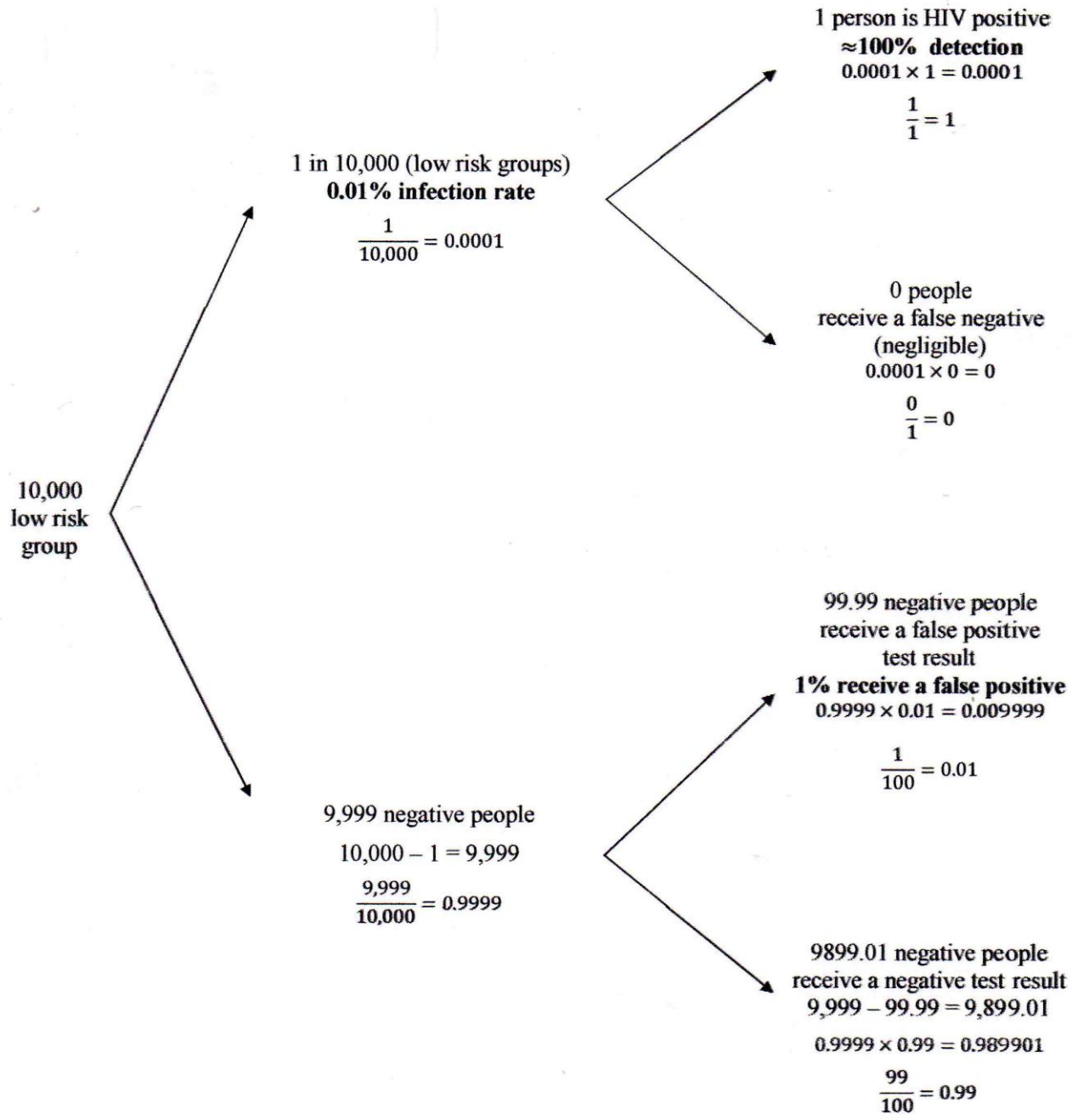
So the infection rate in a population alters the number of false positives:
1 in 10,000… for every 1 correctly diagnosed, 100 people receive a false positive and worry;
5 in 10,000… for every 1 correctly diagnosed, 20 people receive a false positive and worry;
200 in 10,000… for every 2 correctly diagnosed, ≈1 person receives a false positive and worries.

This test works if you are HIV positive, but if you are negative you have a 1% chance of the test giving you a false positive. But the higher the risk in the population, the fewer false positives. It is still 1%, but it is 1% of a lower number (because the infection rate is higher).

I think this test is still useful because many parts of the world don't have the type of medical system we have here. It's not always easy to get to a doctor, and not everyone can afford treatment (it's not always free). The question says this is a quick and cheap test, which is important, because its use can be widespread. It is reliable, because if you are positive, it will confirm it (which is what really matters).

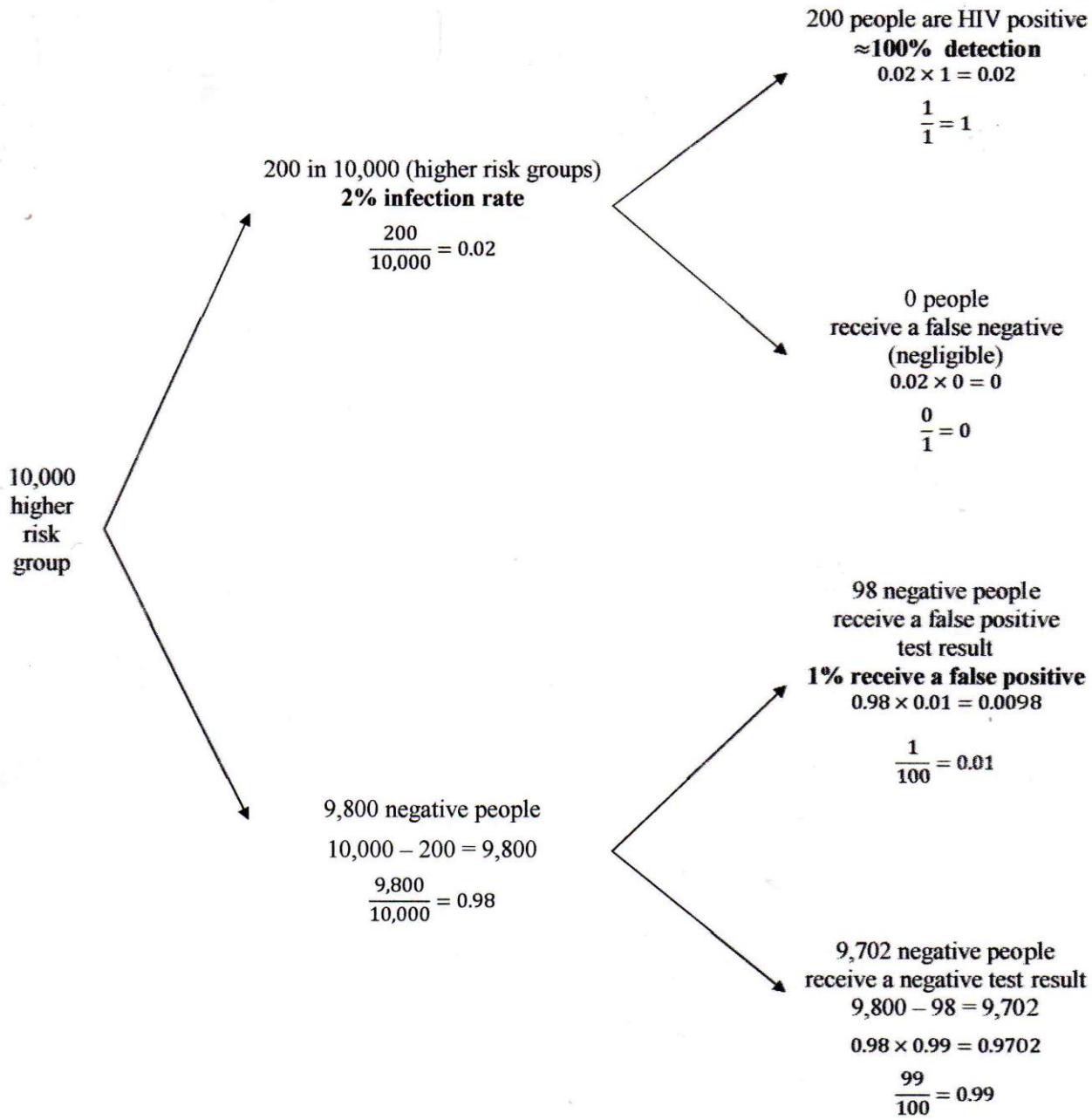
It's almost like a gateway: all positive results would cause worry, but only the true positives have the virus. If you do have the virus, you can try to keep healthy.

I did some diagrams, which do make it much clearer:



So the probability of testing positive and having HIV is:

$$\frac{0.0001}{0.0001 + 0.009999} = 0.0099$$



So the probability of testing positive and having HIV in a higher risk group (200/10,000) is:

$$\frac{0.02}{0.02 + 0.0098} = 0.67$$